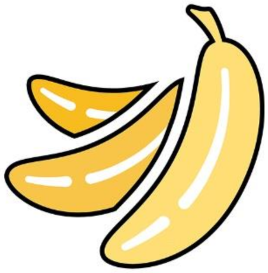
Natural Packages



As part of lunch the other day, I had a banana. Peeling the fruit and composting the peel reminded me of an investigation I first encountered in a 1987 AIMS publication, *Math + Science: A Solution*. The title of the investigation was "The Big Banana Peel," and one of the published versions of this activity featured an illustration of a partially peeled banana. They asked, "What part of a banana is edible?" Well, I know that in some cultures, the peel is used to make more food, so very little is wasted. But I'm not there yet, and for me the answer is simple: The part that is edible is the inside part, right? The activity steers the learner toward a more thoughtful, mathematical, and scientific answer. Students are directed to weigh the whole banana, peel it, and weigh the edible part as well as the peel itself, then compose the fraction:

$$\frac{mass\ of\ the\ edible\ part}{mass\ of\ the\ whole\ thing}.$$

and thus obtain the percentage that is edible. As you can see, the closer the mass of the edible part comes to the mass of the whole thing, the closer this fraction will come to 1 and thus the percentage will approach 100%. So, we will define this percentage as the packaging efficiency.

Appealing Packages

This line of inquiry can go a long, long way. Bananas lead to apples, oranges, grapes, and peanuts, which point to hard-boiled eggs and so forth. And if nature can package things, so can people. Are our packages as efficient as those of nature?

But let's stick with the natural packages for a while. I peeled that lunchtime banana and found that although the whole thing weighed 122 grams, the fruit itself, the edible part, was only 77 grams. Composing the ratio that leads to percentages, I got $\frac{77}{122} = \frac{\%}{100}$. So, $0.631 = \frac{\%}{100}$, and thus the packaging efficiency of that banana is about 63%. Are they all that way? No, a second banana comes in at 68%. Considering my history with bananas, I am intrigued by the idea of repeating this investigation after a banana has had some time to sit around and develop a few brown spots. My experience is that the peel loses its mass as the fruit ages, so will this send the efficiency ratio and thus the percentage up? I set aside a couple of bananas and move on to oranges. Are they more efficiently packaged than bananas?

The first one I try is a navel orange, and the peel is thick. The whole orange before peeling weighs 374 grams, and the edible part weighs 240 grams. That gives me a packaging efficiency of about 64%, remarkably similar to the bananas. Then I try a little mandarin-type orange, one of those that are so popular in plastic net bags these days. The whole thing weighs only 98 grams, and the peel, much easier to remove than that of the navel orange, weighs 27 grams. So, the packaging efficiency for this orange is a little more than 72%. The next one of these I try comes in at only 68%, so some variation exists. With time and a healthy appetite, I could generate some means and standard deviations.

Fig. 1 Natural Packages

Instead I turn to peanuts. A one-pound bag of in-the-shell, roasted and salted peanuts provides some interesting diversions. The basic process is the same, of course: weigh the whole peanut, remove the shell, and weigh the two nuts within to get the packaging efficiency. (They are actually legumes so *nuts* is a misnomer, but I am going with it.) There are some interesting twists, though, as I run into the smaller shells that hold a single nut inside, and only rarely, I find larger shells that hold three nuts—a triple. How will the packaging efficiency change? Looking at the way the shells are structured, I predict that the higher the number of nuts inside, the higher the efficiency of the packaging. And that's how it turned out, but the path to this conclusion was interesting. With the equipment I had, a simple postal scale sensitive to the nearest gram (see figure 1), the shell of a peanut bearing only one nut was too light to register. My solution was to find 10 of them and weigh them in a batch. The result in terms of the packaging efficiency expressed as a percentage now accounts for 10 instances and is therefore a bit more reliable than if I based my analysis on a sample of 1 nut. I do the same for 10 doubles and for the 2 triples I was able to find in the bag. Table 1 shows the results, which make me pine for more triples and some quadruples to test.

Table 1 Natural Peanut Packaging Efficiencies

1 peanut per shell, $N = 10$	62%
2 peanuts per shell, $N = 10$	66%
3 peanuts per shell, $N = 2$	80%

Human Packages

This work with the peanuts reminds me that we have an unopened jar of peanuts in the kitchen cabinet. Humans can package peanuts too! The full plastic jar weighs 507 grams. I unscrew the lid, remove the tamper-proof barrier, and dump out the contents, which weigh 458 grams. So, people have packaged these peanuts with just over 90% efficiency. That beats the peanut shells. It makes me wonder, though. On the basis of my little bit of data, I note a clear trend toward higher efficiencies with more peanuts in the shell. A quick spin around the internet tells me that shells holding more than five nuts are rare indeed, so I suppose I will never find one that holds the hundreds of nuts I poured out of the jar. But if I could and if the (nonlinear) trend established in Table 1 continued, surely the efficiency would be up near the 100 percent mark.

Humans package all kinds of things. Next, I take a look at oranges. We do not package oranges in jars the way we do peanuts, but we do package orange juice. Lots of it. So, I am off to the grocery store to get two containers—plastic again—of OJ. One holds 11.5 ounces, or 340 milliliters; the other holds 52 ounces, or 1,530 ml. This is a little like the small orange and the large one, but both plastic containers seem to be of about the same thickness, so I am not anticipating the decrease in packaging efficiency seen in the oranges as we go from the small container to the large one. It may be the opposite, in fact, as I think that the ratio of surface area to volume—which is related to the ratio of packaging to product—will decrease as the package is scaled up. The shape of the OJ container is not simple but, if it were a simple cylinder, then the ratio of surface area (SA) to volume would be given by $\frac{\text{SA}}{\text{Vol}} = \frac{2\pi r^2+2\pi rh}{\pi r^2 h}$, where r is the radius and h is the height. And in the simple case where $h = r$, this reduces to $\frac{4}{r}$, so that as the size of the container (determined by r) increases, the ratio of surface area, related to the packaging, falls in comparison with the volume, which is related to the product inside. Simply put, the larger the container, the more contents within it in comparison with the surface area of the container, at least for containers like cylinders, cubes, and spheres. I play around with this in Desmos (see figure 2) for a few minutes and then move on to the actual measurements for the two orange juice containers. As it worked out, the 340 ml

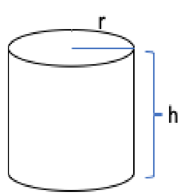
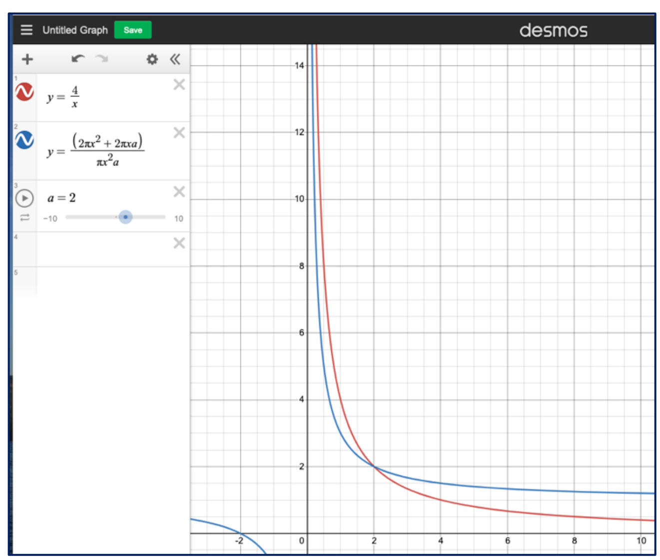


Fig. 2 The Ratio of Surface Area to Volume for Cylinders



bottle had an efficiency rating of 92%, whereas the 1,530 ml bottle was about 96%.

What about raisins? People package raisins in cartons large and small, and the small ones may be found in students' lunches, which makes them an interesting subject for this investigation. One of the really small half-ounce boxes of raisins weighs in at only 16 grams. The raisins themselves weigh 14 grams, and so the packaging efficiency is up near 88%. If my thinking is right about the way packaging efficiency increases as the size of the package increases in human packaging, many of which are right prisms, a big box of raisins should have a higher efficiency percentage than the small one. Investigation of a 12-ounce box of raisins supports this. That package is a little more than 93% product. Composing the same ratio of surface area to volume and considering the simplest box, a cube, we see that $\frac{\text{SA}}{\text{Vol}} = \frac{6s^2}{s^3} = \frac{6}{s}$. And so once again, as the length of the side, s, increases, the ratio of surface area to volume decreases.

As a final case for now, consider the lowly toaster pastry. They come in pairs, and each pair is housed in a plastic/foil bag that weighs next to nothing. The pair taken together weighs 101 grams, so even assuming the bag weighs just under half a gram—which seems like the most it could be, given that it does not register on my scale with 1-gram sensitivity—the efficiency of this packaging is about 99.5%. Wow! …. Maybe not. They come four bags to a box, so the weight of the box (30 grams) that holds the four bagged pairs must be considered as well. That brings them down to about 92%. Still, looking at the results I have obtained for human-made packages versus natural packages, as shown in Table 2, I conclude that at least for this small sample of items, humans win in terms of efficiency when measured by weight.

But wait! Is our simple definition of efficiency sufficient? When I think about the nature of these packages and the resources that went into their production, I decide that there is clearly more to consider here. So much of the packaging we buy these days is plastic, and the resources needed to produce the OJ bottles, for example, are considerable. Another consideration: When I put the banana peel into the compost or even into a landfill, it will eventually become a productive part of the soil. Not so for the plastic bottle. Kind of the opposite, in fact. These things are not accounted for in my simple calculations, but they provide food for thought.

Table 2 Natural vs. Human Packaging Efficiencies

Banana	66%
Naval orange	64%
Mandarin orange	72%
Single peanut	62%
Double peanut	66%
Triple peanut	80%
(Unweighted) Mean for Natural Packages:	**68.3%**
Peanuts in a plastic jar	90%
Orange juice (small bottle)	92%
Orange juice (large bottle)	96%
Raisins (small box)	88%
Raisins (large box)	93%
Toaster pastry	92%
(Unweighted) Mean for Human-Made Packages:	**91.8%**

Conclusion

Oranges and orange juice, peanuts and toaster pastries—maybe I took this a little further than would be expected, but I enjoyed it, I learned a few things, I thought some new thoughts, I used math as a tool to make sense of the world I live in, and when I find that quadruple peanut, I'll be back. But hark, what's that I hear? A cardboard box being dropped on our front porch and a truck pulling away? Another package?

Lesson Plan

Learn more about implementing Natural Packages in your classroom by exploring the Illuminations lesson here! Then, share your experiences using Math Sightings on social media with the hashtag #MathSightings.

References

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